Favoritism and Anonymity in Auctions

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Motivation

- In many auctions, there is *favoritism*: the seller (or buyer) cares about the welfare of a subset of the set of bidders.
  - Domestic vs. foreign bidders in public procurement.
  - Small firms.
  - Firms in the same economic group.

Motivation II

- Favoritism often motivates the use of *discriminatory auctions.*
- When is an auction discriminatory?
  - When it is not an anonymous mechanism: not all bidders are treated equally.
  - It happens in practice (e.g. price preferences, rights of first refusal).
  - There may be reasons other than favoritism to use nonanonymous auction formats (e.g. revenue maximization with asymmetries).
- General questions:
  - Are the usual ways of discrimination adequately justified by favoritism? What's the optimal auction when there is favoritism? Is there scope for favoritism when discrimination is not allowed?

Setting I

- Favoritism in auctions
  - **Multidimensional** auctions (auctioneer assesses product quality): Laffont and Tirole (1991) and Vagstad (1995)
  - **Single-dimensional** auctions (price-preferences may be used): McAfee and McMillan (1989), Branco (1994) and Naegelen and Mougeot (1998)
- We focus here on the single-dimensional case.

Setting II

- One unit of an indivisible object is sold through an auction.
- Value of the object for the seller: zero.
- Initially, there are N potential buyers.
- Value of the object for bidder i: v_i, her private information.
- v_i distributed according to the c.d.f. F_i, with support $\left[\underline{v}_i, \overline{v}_i\right]$ and a density f_i that is positive and bounded on the whole support. Bidders' valuations are independent.
- We have the simplest possible setting: independent private values.
- The seller and all bidders are risk neutral.

Roadmap

1. The optimal auction when the seller values the welfare of some bidders
2. A practical way to favor a bidder: the right of first refusal
3. Favoritism with an endogenous number of bidders
4. Favoritism and Anonymity

The Optimal Auction with Favoritism

- In the auction, each bidder i chooses a bid b_i. The auction rules specify, for each vector of bids, which bidder wins and how much each bidder pays.
  - What rules are optimal under favoritism?
- The seller cares about the welfare of a subset of bidders
- Our take: he should maximize a weighted sum of her revenue and the favored bidders' expected utility
- To simplify, assume that
  - There is only one favored bidder: wolog, it is bidder 1.
  - The seller maximizes joint surplus: bidder 1's welfare carries the same weight as the seller's revenue.
  - We get a particular case of Naegelen and Mougeot (1998).

The Optimal Auction with Favoritism II

- Let
  - $H_i(v_1, ..., v_N)$ be the probability that bidder i gets the object,
  - $P_i(v_1, ..., v_N)$ be the price bidder i has to pay to the seller,
  - $h_i(v_i)$ be the expected probability that bidder i gets the object when her valuation is v_i,
  - $p_i(v_i)$ be the expected price she pays when her valuation is v_i.
- Bidder i's expected utility when her valuation is v_i is

$$U_i(v_i) = h_i(v_i)v_i - p_i(v_i)$$

The Optimal Auction with Favoritism III

- The Seller's problem is

$$\max_{\{H_i(.),P_i(.)\}_{i=1}^N} \sum_{i=1}^{N} \int_{\underline{v}}^{\overline{v}} p_i(v_i) f_i(v_i) dv_i + \int_{\underline{v}}^{\overline{v}} U_1(v_1) f_1(v_1) dv_1$$

subject to

$$\begin{aligned} U_i(v_i) &\geq \widetilde{U}_i(v_i, v_i') \quad \text{for all } i, \text{ for all } v_i, v_i' \\ U_i(v_i) &\geq 0 \qquad\qquad\qquad \text{for all } i, \text{ for all } v_i \end{aligned}$$

where $\widetilde{U}_i(v_i, v_i') = h_i(v_i')v_i - p_i(v_i')$

The Optimal Auction with Favoritism IV

- The allocation rule that maximizes joint expected surplus is then

$$H_1(v_1, ..., v_N) = \begin{cases} 1 & \text{if } v_1 > \max_{i \neq 1} J_i(v_i) \\ 0 & \text{otherwise} \end{cases}$$

$$(\text{for } i > 1) \quad H_i(v_1, ..., v_N) = \begin{cases} 1 & \text{if } J_i(v_i) > \max\{v_1, \max_{j \neq i} J_j(v_j)\} \\ 0 & \text{otherwise} \end{cases}$$

where $J_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ is bidder i's *virtual valuation*.

The Optimal Auction with Favoritism V

- A form of discrimination: the favored bidder's *actual* valuation is compared to the unfavored bidders' *virtual* valuations. Remember $v > J_i(v)$.
- Expected payments follow from incentive compatibility.
- This form of discrimination, of course, remains even under symmetry (i.e. when $F_i(v) = F(v)$ for all i)
- How do we implement this?

The Right of First Refusal

- A practical form of favoritism: ROFR clauses are broadly used in share transactions, lease contracts, partnerships and professional sports, among many other cases (Walker, 1999).
- The favored bidder has the right to match the highest bid made by an unfavored bidder and win.
- An ROFR clause can be added to any auction.
- Given its use in practice, there is a literature on the ROFR.
- Choi (2009), Burguet and Perry (2009), Lee (2008) and Bikhchandani, Lippman and Ryan (2005) show that **there are cases where adding an ROFR clause could increase the joint expected surplus** for the seller and the favored bidder.

May the ROFR be an optimal form of discrimination?

- Our take (Arozamena and Weinschelbaum, 2006): **could an ROFR clause be part of a mechanism that maximizes the expected joint surplus** of the seller and the favored bidder?
- The favored bidder will match the "standing bid" and win whenever that standing bid is below v_1.
- If a mechanism with an ROFR clause maximized joint expected surplus, then, the standing bid would always have to be the highest among the unfavored bidders' virtual valuations.

May the ROFR be an optimal form of favoritism?

- For all $i > 1$, $P_i(v_1, ..., v_N) = J_i(v_i)$ whenever bidder i gets the object.
- Assume that only the winning bidder pays (just to simplify, the argument extends to the general case)
- Then,

$$p_i(v_i) = h_i(v_i)J_i(v_i)$$

- But before we had that, by incentive compatibility,

$$p_i(v_i) = h_i(v_i)v_i - \int_{\underline{v}}^{v_i} h_i(s)ds$$

- Are these two compatible? No, since in general

$$\int_{\underline{v}}^{v_i} h_i(s)ds \neq h_i(v_i)\frac{1 - F_i(v_i)}{f_i(v_i)}$$

Entry: A Key Issue

- Klemperer (2002): "...the most important issues in auction design are the traditional concerns of competition policy –preventing collusive, predatory, and entry-deterring behavior."

Entry and Favoritism

- So far, we have examined favoritism in auctions under the assumption that the number of bidders is fixed.
- Intuitively, if participating in an auction is costly, favoring some bidders may discourage those who are not favored from entering.
- How does endogenizing the number of bidders alter the optimal auction under favoritism?
- First, we have to model the entry decision made by potential buyers.

Two Alternative Ways to Model Entry

- It is always assumed that participating entails some cost.
- But when is the entry decision made?
  - Entry decision is made **after** knowing the valuation. Samuelson (1985), Stegeman (1996); Menezes and Monteiro (2000); Celik and Yilankaya (2009) and Li and Zhen (2008).
  - Entry decision is made **before** the valuation is realized (our assumption). McAfee and McMillan (1987) and Engelbrecht-Wiggans (1987, 1993) Levin and Smith (1994) and Ye (2004).

Entry and Favoritism II

- As with a fixed number of bidders, endogenous entry may lead to discriminatory auctions even if the seller only maximized her own revenue.
- A new reason for that:
  - Now weak bidders are less likely to enter, so favoring them may raise N and enhance competition in the auction.
- Again, we are interested in what happens when the seller actually maximizes her and the favored bidders' joint surplus (Arozamena and Weinschelbaum, 2011).
  - To abstract from other motivations for discrimination, we assume symmetry.

The Modified Setting

- Large number of potential bidders, N. Each of them can take part in the auction by paying a symmetric, fixed cost $k \geq 0$.
- Bidder i learns her valuation v_i after entering.
- v_i distributed according to the c.d.f. $F_i = F$ for all i.
- Again, assume a single preferred bidder (bidder 1).

The Seller's Problem

- First, the seller announces a mechanism, then bidders decide whether to enter or not.
- Let $\mathbf{B} \subseteq \{1, ..., N\}$ be the set of bidders that enter the auction. The seller may condition the mechanism on $\mathbf{B}$.
- The seller announces functions

$$H_i^{\mathbf{B}}((v_i)_{i\in\mathbf{B}}), \quad P_i^{\mathbf{B}}((v_i)_{i\in\mathbf{B}}), \quad \mathbf{B} \in 2^{\{1,\dots,N\}}$$

- Bidder i's expected utility, when $i \in \mathbf{B}$, her valuation is v_i and she expects all $j \neq i$, $j \in \mathbf{B}$ to enter is

$$U_i(v_i, \mathbf{B}) = h_i(v_i, \mathbf{B})v_i - p_i(v_i, \mathbf{B})$$

where $h_i(v_i, \mathbf{B}) = E_{v_{-i}}[H_i^{\mathbf{B}}((v_j)_{j\in\mathbf{B}})]$ and $p_i(v_i, \mathbf{B}) = E_{v_{-i}}[H_i^{\mathbf{B}}((v_j)_{j\in\mathbf{B}})]$, $i \in \mathbf{B}$.

The Seller's Problem II

$$\max_{\substack{H_1^{\mathbf{B}}(v_1,\ldots,v_n),P_1^{\mathbf{B}}(v_1,\ldots,v_n) \\ \mathbf{B}\in 2^{\{1,\ldots,N\}}}} \sum_{i\in\mathbf{B}^*} \int_{\underline{v}}^{\overline{v}} p_i(v_i,\mathbf{B}^*)f(v_i)dv_i + \int_{\underline{v}}^{\overline{v}} U_1(v_1,\mathbf{B}^*)f(v_1)dv_1 - k$$

subject to

$$\begin{aligned} U_i(v_i,\mathbf{B}) &\geq h_i(v_i',\mathbf{B})v_i - p_i(v_i',\mathbf{B}) && \text{for all } \mathbf{B}, \text{ for all } i\in\mathbf{B}, \text{ for all } v_i, v_i' \\ U_i(v_i,\mathbf{B}) &\geq 0 && \text{for all } \mathbf{B}, \text{ for all } i\in\mathbf{B}, \text{ for all } v_i \end{aligned}$$

$$\int_{\underline{v}}^{\overline{v}} U_i(v_i,\mathbf{B}^*)f(v_i)dv_i \geq k, \forall i\in\mathbf{B}^*$$

$$\int_{\underline{v}}^{\overline{v}} U_j(v_j,\mathbf{B}^*\cup j)f(v_j)dv_j \leq k, \forall j\notin\mathbf{B}^*$$

The Optimal Mechanism with Endogenous Entry

- Whith free entry of bidders the optimal auction format changes substantially.

Proposition

The allocation rule that maximizes joint expected surplus is a non discriminatory one. A first price auction with reservation price equal to the seller's valuation that treats equally the favored and the non favored bidders maximizes joint surplus.

Proof

- Without favoritism, expected value of the game = winning bidder's expected valuation - total entry cost.
- That equals expected revenue (i.e. the seller's surplus) plus all bidders' surpluses
- The allocation rule that maximizes the expected value of the game is to allocate the good to the entrant with the maximum valuation irrespective of her identity.
- **The first price auction with no reservation price maximizes the expected value of the game. The seller gets the whole surplus.**
- We need to prove that when the seller cares about bidder 1's surplus she cannot do better. If this were the case she would be getting more than the maximal value of $Ev - \#(\mathbf{B}^*)k$. At least one of the bidders has to receive a negative expected surplus, absurd.

Remarks

- The seller does not care if the favored bidder is in the set of entrants or not -he can subsidize the prefered bidder to induce him to enter and still get the same surplus.
- But if there is discrimination among entrants, total surplus decreases, and so does the utility of the seller.
- The same result holds for the case of symmetric, mixed-strategy equilibria, as in Levin and Smith (1994).
- ROFRs are not optimal. With an ROFR (if it matters) the bidder with the highest valuation does not win. Thus there is ineficiency.

Anonymous Auctions

- Back to N fixed.
- Discrimination may not be possible
  - Public procurement laws and regulations frequently forbid discrimination. For example, higher-level regulations may explicitly prevent local authorities to favor local firms.
  - This constraint may be interpreted as imposed by a principal on an agent who is in charge of the auction.
- What if there is favoritism but the seller is not allowed to discriminate?
  - A particular case is analyzed in Arozamena, Shunda and Weinschelbaum (2014)
  - v_i distributed according to the c.d.f. $F_i = F$ for all i (we focus on symmetric equilibria)

The Seller's Problem

- As before, assume bidder 1 is the favorite (this is generalized in the paper). Let $\alpha \in [0,1]$ be the weight that the seller attaches to bidder 1's profits in her own objective function.
- Then, as above, her problem is

$$\max_{\{H_i(.),P_i(.)\}_{i=1}^N} \sum_{i=1}^N \int_{\underline{v}}^{\overline{v}} p_i(v_i) f(v_i) dv_i + \alpha \int_{\underline{v}}^{\overline{v}} U_1(v_1) f(v_1) dv_1$$

subject to the incentive-compatibility and participation constraints

$$\begin{aligned} U_i(v_i) &\geq \widetilde{U}_i(v_i, v_i') \quad \forall\ i, \forall\ v_i, v_i' \\ U_i(v_i) &\geq 0 \qquad\qquad\quad \forall\ i, \forall\ v_i \end{aligned}$$

- Given that the seller *cannot* discriminate among bidders, there's a new constraint.

The Seller's Problem II

- *Mechanism* $\{H_i(.), P_i(.)\}_{i=1}^{N}$ *has to be invariant to permutations*

For any permutation $\pi : \{1, ..., N\} \longrightarrow \{1, ..., N\}$ $\forall(v_1, ..., v_N)$, *satisfies*

$$\begin{aligned} H_i(v_{\pi(1)}, ..., v_{\pi(N)}) &= H_{\pi(i)}(v_1, ..., v_N) \\ P_i(v_{\pi(1)}, ..., v_{\pi(N)}) &= P_{\pi(i)}(v_1, ..., v_N) \end{aligned}$$

- This implies that the seller's problem can be expressed in terms of order statistics: she has to choose an allocation function

$$\{H_n(v_{(1)}, ..., v_{(N)})\}_{n=1}^{N}$$

The Seller's Problem III

- We can focus only on which allocations the seller chooses when the vector of valuations is ordered (allocations in all other cases follow from the no-discrimination constraint)
- The probability any bidder wins has to depend only on
  - (i) the vector of order statistics and
  - (ii) her valuation's position in that vector
- But the seller cares about the identities of the bidders.
- Since valuations are i.i.d., the probability that bidder i's valuation ranks in any position n is the same of all bidders $(1/N)$.

The Seller's Problem IV

- The seller's objective function becomes

$$\sum_{n=1}^{N} H_n(v_{(1)}, ..., v_{(N)}) \left[v_{(n)} - \frac{1 - F(v_{(n)})}{f(v_{(n)})}(1 - \frac{\alpha}{N}) \right]$$

- $v_{(n)} - \frac{1-F(v_{(n)})}{f(v_{(n)})}(1 - \frac{\alpha}{N})$ takes its highest value for $n = 1$. (Since $J(v) = v - \frac{1-F(v)}{f(v)}$ is increasing).

Theorem

The optimal allocation rule is

$$H_i(v_1, ..., v_N) = \begin{cases} 1 \text{ if } v_i > \max_{j \neq i} v_j \text{ and } v_i - \frac{1-F(v_i)}{f(v_i)}(1 - \frac{\alpha}{N}) > 0 \\ 0 \text{ if not} \end{cases}$$

Implementation

- This direct mechanism can be implemented by any efficient auction with an adequately chosen reserve price or entry fee.
- For example, the seller may choose a first-price or a second-price auction with reserve price r such that

$$r - \frac{1 - F(r)}{f(r)} \left(1 - \frac{\alpha}{N}\right) = 0$$

- If $\alpha = 0$, the optimal mechanism is the standard, revenue-maximizing direct mechanism: the object is awarded to the highest-valuation bidder and all valuations below r such that $r - \frac{1-F(r)}{f(r)} = 0$ are excluded.

Remarks

- r (the the minimum valuation that is not excluded) is decreasing in α.
- Results are "robust" to other forms of favoritism
- Seller's utility= revenue + $\sum_i w_i$.(prob. that i wins)
  - where w_i is how much the seller values that buyer i gets the object
  - The reserve price is

$$Nr - N\frac{1 - F(r)}{f(r)} + \sum_{i=1}^{N} w_i = 0$$

Does preventing discrimination raise expected revenue?

- We have shown it does in some cases:
  - With $N \geq 2$, and for any $F(v)$, if $\alpha = 1$.
  - With $N = 2$, for any α, when $f(.)$ is differentiable, $F(.)$ has monotone increasing hazard rate and $J(v)$ is convex.
    - This holds, for example, with power function distributions.

Anonymous Mechanisms

- Is anonymity costly to the seller?
- Deb and Pai (2013) have found that it is not.
  - But their mechanisms could be very complicated.
  - In some cases they have to select equilibria in a very specific way for them to be able to replicate, under anonimity, what the seller can do without requiring that condition.
- Arozamena, Francetich, Manelli and Weinschelbaum (in progress) examine the interaction of two conditions (anonymity and dominant strategy implementation) under independent private values and risk neutrality.

Anonymous Mechanisms II

- We know from Manelli and Vincent (2010) that demanding dominant strategies has no effect as to what we can implement (as compared to Bayesian implementation).
- We know from Deb and Pai that demanding equal treatment of agents (anonymity) in the indirect mechanism does not constrain the designer.
- What happens when we demand both conditions?
  - The answer is that, "in general," it is costly to the seller.
  - This holds for a revenue-maximizing seller and also for a case of favoritism.